Calculation of Annuity Reserves
From Elementary Values

The single life tables give values of a_x which is the reserve per $1 of annual income for an annuity with the first of equal payments commencing in exactly one year. The joint life tables give values of a_{xy} which is the reserve per $1 of annual income for an annuity with the first of equal payments commencing in exactly one year, and whose payments continue as long as both annuitants are alive.

The ages x and y are those as of the annuitant's nearest birthday.

A joint and survivorship annuity, which pays as long as either annuitant is alive, is given by:

$$a_{\overline{xy}} = a_x + a_y - a_{xy}$$

For annuities which are payable more frequently than annually, the following modal approximations may be used or a company may use mean reserves (see below).

$$a_x^{(m)} = a_x + (m-1)/2m \quad \text{and} \quad a_{xy}^{(m)} = a_{xy} + (m-1)/2m$$

For example, the reserve per $1 annual income for a quarterly annuity (i.e. per 25 cents of quarterly payment) whose next payment is three months away would be $a_x + 3/8$.

The reserves to be set up at issue are calculated by multiplying the above formulas by the annual (or annualized) income.

The following mean reserve factors are multiplied by the amount of annual income to compute reserves as of year-end:

single life mean reserve factor = $\frac{1}{2} * (a_x + a_{x+1} + 1)$

joint and survivor two life mean reserve = $\frac{1}{2} * (a_{\overline{xy}} + a_{\overline{x+1:y+1}} + 1)$ where x and y were ages nearest birthday as of the last anniversary (of the issue date). The formulas for mean reserves need not vary by the number of payments per year, but they must be multiplied by the total annual income.

A single premium immediate annuity is one purchased with one payment (the charitable contribution) whose payments begin within one year of the issue date. Any annuity whose payments begin later is a deferred annuity, which is subject to more general reserve standards. For annuities with starting payout dates that are flexible, a greatest present value approach shall be used, consistent with the requirements in Section 99.4(d) of Regulation 151.

MINIMUM RESERVE CALCULATION
FOR DEFERRED GIFT ANNUITIES

Step 1

Look up the maximum valuation interest rate from Benefit Category F of the Circular Letter. The rates vary by issue year and guarantee duration. The guarantee duration is the number of years from the date of the charitable contribution to the date that the annuity benefits are scheduled to commence. Any rate less than or equal to this rate may be used.

Step 2

Determine s and x where:
s = the number of whole years from the valuation date to the date of next payment and
x = the annuitant's age nearest birthday on the most recent anniversary of the gift.

Step 3

For a fixed age x , $\sum_{t=s}^{w-x-1} v^t \, {}_t p_x$ represents the present value of an annuity payable to an individual age x with the first of equal payments of 1 commencing s years from now.

When s=1 the aforementioned is a_x .

When s=0 the aforementioned is 1+ a_x .

When s= k >0 the aforementioned is then known as $(k-1)| \, a_x$.

Step 4

The single life mean reserve factor is given by

$$.5 * (s| \, a_x + s| \, a_{x+1} + v^s * .5 * ({}_s p_x + {}_s p_{x+1}))$$

When s=0 the aforementioned reduces to ½*($a_x + a_{x+1}$ +1)

where i is the applicable interest rate and where v = 1/(1+i) . For example if i = 8% then v = 1/1.08.

${}_s p_j = l_{j+s} / l_j$ and w is the end age of the mortality table. (see step 3. above). w = 115 for both the 1983 "a" and Annuity 2000 Tables. w = 120 for the 2012 IAR Table.

Step 5

The joint and survivor two lives mean reserve factor is obtained by replacing x by xy and by replacing x+1 by (x+1)(y+1) and ${}_s p_x$ by ${}_s p_x \, {}_s p_y$